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ON THE RELATIONSHIP OF BEAM TRANSMITTANCE ON CLEARNESS INDEX FOR ATHENS, GREECE

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Proper design and performance prediction of solar energy systems requires accurate information on the availability of solar radiation. However, the complexity and cost of a reliable radiation measuring network dictates the need for developing alternatives that can replace and/or complement observations. Dependable solar radiation models are an alternative.

In this paper, two years of solar radiation data from the National Observatory of Athens, Greece (37.5°N, 23.5°E, 107 m above MSL) are used to develop a continuous piecewise linear regression model that predicts hourly direct irradiation values when only global irradiation data are available. The applicability of the Randall–Whitson [7] and Jeter–Balaras [9] models in the area of Athens is also evaluated. For the urban atmospheric conditions of Athens the developed two year correlation provides a better fit to the empirical data.

KEY WORDS: Beam transmittance, Clearness index, Radiation modeling.

1 INTRODUCTION

Reliable solar radiation data are necessary for the successful design of many solar energy systems. For example, simulation programs like TRNSYS [1] require that the user provides data on the available amounts of solar radiation at the area of interest. In particular, researchers are mostly interested in the components of solar radiation – the direct and diffuse radiation. Existing stations around the world usually measure global solar radiation on a horizontal surface. A limited number of stations also measure direct radiation, and diffuse radiation on a horizontal plane. However, the installation of a complete monitoring station at each site of interest is not always possible and other means of attaining this information must be used.

Numerical models that can predict the direct and/or diffuse radiation are an alternative. Such models can use dependable global radiation data to estimate the other solar radiation components. Even when these data are measured at an existing monitoring site, it may be necessary (for compiling long unbroken

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sequences of data) to estimate the components of solar radiation for missing measurements or even for validation purposes of collected data.

Prior work by Lalas *et al.* [2], has produced correlations for the average daily and monthly values of diffuse radiation in terms of global radiation values for Greece. In many applications though, there is a need of direct radiation data on an hourly basis. In this paper a correlation is obtained between hourly direct and global radiation for Athens, Greece.

The relationship is expressed in terms of the irradiations which are the time integrals (i.e. one hour) of the radiant flux or irradiance. A convenient representation is the relationship between two dimensionless numbers, the beam transmittance of the atmosphere τ_b,

$$\tau_b = \frac{I_h}{I_{0h}} = \frac{\dfrac{G - D_h}{\cos \zeta}}{I_{0h}} \tag{1}$$

where I_h = hourly direct irradiation,
I_{0h} = hourly extraterrestrial direct irradiation,
G = hourly global irradiation,
D_h = hourly diffuse irradiation, and
ζ = solar zenith angle,

and the clearness index k_t,

$$k_t = \frac{G}{G_0} \tag{2}$$

where G_0 = hourly extraterrestrial horizontal irradiation.

The hourly values for the extraterrestrial direct radiation in Equation (1) were computed according to the formula:

$$I_{0h} = I_0 (R_{\text{ave}}/R)^2 \, \Delta t \tag{3}$$

where I_0 = solar constant, 1367 W/m^2,
R_{ave} = average earth–sun distance,
R = prevailing earth–sun distance, and
Δt = time interval, 1.0 hour.

The hourly values for the extraterrestrial horizontal irradiation in Equation (2) were calculated according to the following formula:

$$G_0 = (12\,\text{hrs}/\pi)\, I_0\, (R_{\text{ave}}/R)^2 \times \{\cos \Phi \cos \delta\, (\sin \omega_2 - \sin \omega_1) + (\omega_2 - \omega_1) \sin \Phi \sin \delta\} \tag{4}$$

where Φ = latitude,
δ = solar declination,
ω_1, ω_2 = solar hour angle at start, end of hour.

The algorithm for calculating the earth–sun distance, the declination, and the Equation of Time used to compute the hour angles in equations (3) and (4) were based on the so-called "low-precision" formulas in [3] and are similar to the program [4] prepared for precise tracking of paraboloidal dishes. The algorithm was tested by comparison with data from [5] for 1983. The maximum observed errors for the solar declination were 0.006 degrees and for the Equation of Time were 0.057 minutes.

Notable relationships between the hourly beam transmittance and clearness index are the pioneering efforts of Boes *et al.* [6], and the highly regarded results of Randall and Whitson henceforth (R–W) [7]. The R–W correlation between the beam transmittance and clearness index was developed based on composite hourly data from five stations in the United States: Albuquerque, NM; Fort Hood, TX; Livermore, CA; Maynard, MA; and Rayleigh, NC.

A model to predict the direct irradiation from measured global solar radiation is also available by Turner and Mujahid [8]. In their development, statistical regression tests were conducted over a range of solar altitude angles. Based on their model comparisons with annual data sets from five locations around the United States, the R–W model was found to be the best available model.

Jeter and Balaras [9] used five years of data collected at the Solar Total Energy Project site in Shenandoah, Georgia, to develop a correlation between τ_b and k_t, henceforth referred to as the J–B model. The R–W and J–B models are very similar, except in the highest k_t range ($k_t > 0.75$). According to available evidence [9], the J–B model is to be preferred in the area of disagreement.

In this paper, the analysis and the model development, similar to the one presented by Jeter and Balaras [9], are based on hourly solar radiation data collected at the National Observatory of Athens (37°59′N, 23°45′E, 107 m above MSL) for the period from April 1983 to May 1985. The performance of the widely accepted Randall and Whitson model [7] and the Jeter and Balaras model [9] in predicting hourly direct solar radiation for the area of Athens, is also evaluated.

2 DATA BASE

The National Observatory of Athens has made available measured hourly values of global and diffuse solar irradiation on a horizontal surface in KJ/m^2, for a period of two years. The measurements were made with Eppley PSP pyranometers using a shadow band to measure diffuse irradiation, for which all appropriate corrections have been incorporated. The hourly direct radiation values, and subsequently the beam transmittance, were calculated through Equation (1). The data base is also composed of the corresponding clearness index values, calculated through Equation (2).

The development of an empirical correlation demands a reliable underlying data base. A data quality control procedure was used to identify and eliminate any erroneous measurements from the available data. Data from predominantly overcast days and data for sun elevations less than six degrees were excluded. Consequently, the remaining 5292 hourly values constitute the two-year data set used in our model development.

3 MODEL DESCRIPTION

The two year solar radiation data from Athens were used to develop a correlation between the beam transmittance and clearness index. The τ_b and k_t data were grouped into eight clearness index bands (that range between $0.05 < k_t < 0.85$) for consistency with the R–W and J–B studies. A homogeneous least squares fit for each band of data was then performed to produce a piecewise linear regression.

Table 1. Annual and two year regression coefficients based on data from the national observatory of Athens

Clearness index interval	*Annual models APR83-MAR84 (3013 cases)*	*APR84-MAY85 (2279 cases)*	*2-YR Model APR83-May85 (5292 cases)*
$0.00<k_t<0.05$	0.0000	0.0000	0.0000
$0.05<k_t<0.15$	0.1335	0.1019	0.1142
$0.15<k_t<0.25$	0.1836	0.4046	0.3015
$0.25<k_t<0.35$	0.3997	0.2286	0.2970
$0.35<k_t<0.45$	0.8086	1.2990	1.0059
$0.45<k_t<0.55$	1.1493	0.6815	0.9806
$0.55<k_t<0.65$	1.1666	1.6219	1.3557
$0.65<k_t<0.75$	1.4252	1.1361	1.2876
$0.75<k_t<0.85$	0.6850	0.4067	0.6167

For each band, the regression model which assures continuity is,

$$(\tau_{bj} - y_{0i}) = \beta_i(k_{tj} - x_{0i}) + \varepsilon_j \quad (5)$$

where x_{0i} = lower limit for band i,
y_{0i} = regression model at x_{0i},
β_i = regression coefficient, and
ε_j = residual error for data (k_{tj}, τ_{bj}).

Results of the linear regression analysis are given in Table 1. The regression coefficients are given for each year and for the whole two year period. The two year data and the developed two year regression model are shown in Figure 1,

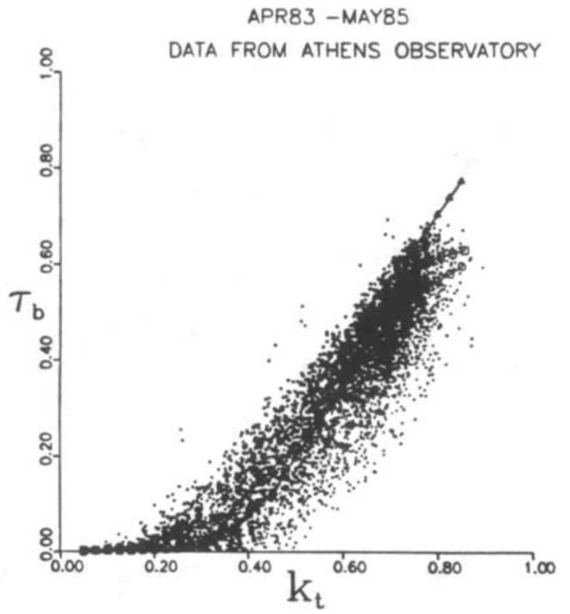

Figure 1 Scatter of and piecewise regression on data from the National Observatory of Athens for the two year period (March 1983–May 1985). The Randall and Whitson Model [7] is identified with squares. The Jeter and Balaras Model [9] is identified with triangles.

superimposed along with the R–W and J–B models. The number of available data increases with increasing value of clearness index, reaching a maximum around (0.65–0.75) with 1834 hourly periods. This is a direct result of the long periods of clear skies in the area of Athens when high values of clearness indices are observed.

The accuracy to which the direct radiation values can be determined from these correlations is highly dependent on the precision of the global measurements. Any error in the global radiation data translates into twice the error from the calculated direct radiation values, because to a close approximation the direct radiation value is proportional to the square of the global value. This can be verified by reference to Figure 2 and by noting the relative magnitude of corresponding τ_b and k_t values. Also shown in Figure 2 are the R–W and J–B models. We may observe that both models underestimate the beam transmittance for low clearness indices, and overestimate it for higher clearness indices. A possible explanation for this behavior is in connection to the characteristic atmospheric conditions of the region, which are briefly described in the following text.

A monthly analysis of the data revealed some useful information about the data. First, the least amount of observations were recorded during the winter months, with mostly uniform distribution of low τ_b values for intermediate cases of k_t. This is primarily the behavior one would expect for winter months in a Mediterranean climate, during which mostly cloudy skies prevail. Second, for the remaining months the opposite is true. There is a persistent high concentration of data at high clearness indices. This is directly related to the predominance of clear skies during these months.

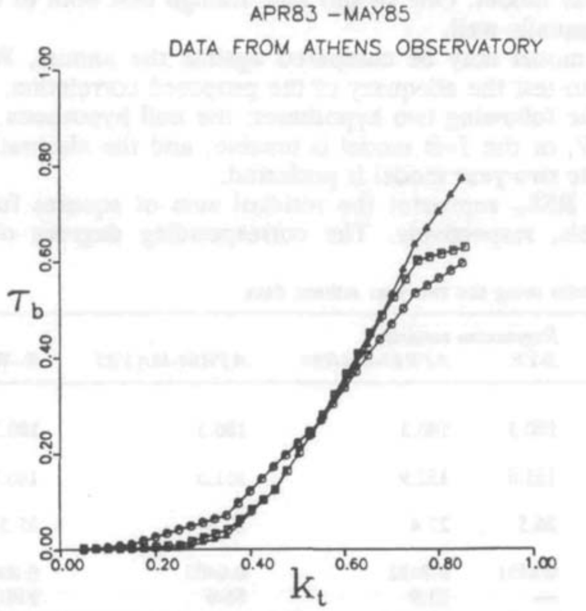

Figure 2 The two year (March 1983–May 1985) regression model based on data from the National Observatory of Athens (identified with polygons) compared with the Randall and Whitson model [7] (identified with squares) and the Jeter and Balaras model [9] (identified with triangles).

For the area of Athens, though, the summer months exhibit another distinct characteristic – unusually low values of beam transmittance at high clearness indices. This is probably due to the existence of significant atmospheric pollution that prevails during the summer months in the city of Athens. That of course would result in an unusually high value for the diffuse irradiation, and low I_{bn}, for high values of global irradiation. These observations were also made by Lalas *et al.* [2].

For the last band, between (0.75–0.85) the number of available data is reduced substantially. Thus, for these high clearness indices any questionable data can easily bias the model. The regression coefficient for this band appears not to be following the trend set by the previous band, which for all three cases is consistently higher (Table 1).

4 MODEL COMPARISONS

To determine how well the proposed relationships can describe the data, one can perform a number of statistical tests. The coefficient of determination (r^2) is often used to judge the adequacy of a regression model. It is defined as the ratio of the residual (RSS) to total (S_{yy}) variation, and it is a measure of how much variation in the beam transmittance is accounted for by the piecewise linear dependence on the clearness index established in the analysis.

The results of an analysis of variance are summarized in Table 2. It is clear from this table that the correlation of the beam transmittance on clearness index is positive, but not equal to unity. Thus, only part of the observed variation can be accounted for by the single regression function of the form $\tau_b = f(k_t)$. Based on the available data, the highest (85%) coefficient of determination is given by the 83–85 two-year model. One should note though that both of the two annual models perform equally well.

The two-year model may be compared against the annual, R–W, and J–B models in order to test the adequacy of the proposed correlation. An F-test was applied to test the following two hypotheses: the null hypothesis, that either an annual, the R–W, or the J–B model is tenable, and the alternative hypothesis, that the composite two-year model is preferred.

Let RSS_0 and RSS_H represent the residual sum of squares for the null and alternative models, respectively. The corresponding degrees of freedom are

Table 2. Statistical results using the two year Athens data.

Statistical variable	*Regression models* 2-YR	*APR83-MAR84*	*APR84-MAY85*	*R–W*	*J–B*
Total variation (S_{yy})	180.3	180.3	180.3	180.3	180.3
Explained variation (ESS)	153.8	152.9	151.5	145.0	134.6
Residual variation (RSS)	26.5	27.4	28.8	35.3	45.7
Coefficient of Determination (r^2)	0.8531	0.8482	0.8405	0.8045	0.7446
F-statistic	—	21.9	56.6	218.5	479.8

designated by df_0 and df_H. Then, the difference $RSS_0 - RSS_H$ represents the additional amount of scatter that was explained by the alternative model (or the null hypothesis if RSS_0-RSS_H is less than zero). Consequently, the degrees of freedom are reduced by $df_0 - df_H$.

The F-statistic, which is the ratio of the mean sum of squares for the regression (MS_R) to the mean sum of square for the residual (MS_H), follows an F distribution with $(df_H - df_0, df_H)$ degrees of freedom. When the alternative model fits the data better than the null model, then the value of RSS_H is less than RSS_0. The hypothesis testing is governed by a comparison of the calculated value of the F-statistic with tabulated values of $F(\alpha, df_H - df_0, df_0)$, where α is the confidence interval.

Results are given in Table 2. The alternative hypothesis is found to be true, with confidence of 99%, for large values of the F-statistic relative to F (0.01, 8, 5292) which corresponds to a tabulated value of 2.64. The large values of the F-statistic for every hypothesized model tends to support the two-year model as a better alternative.

Hourly correlations of the form $\tau_b = f(k_t)$ may perform inadequately for a particular hour on a given day. However, in the case that hourly models are to be used for performance simulations over longer periods, such as a month or a year, we should expect that both positive and negative errors would average out to give good long-term results. Accordingly, each one of the three models (the Athens two-year model, the Randall Whitson model, and the Jeter Balaras model) were used to compare monthly averages of predicted and recorded hourly values of τ_b using the 1984 data base from Athens.

The percent errors produced by each of the models on a monthly basis are shown in Table 3. According to the annual averages, the Athens two-year model exhibited the best performance (less than 1%), with the R–W and J–B models predicting τ_b within about 6% of the recorded values. The hourly maximum or minimum errors for a given month, were consistently smaller in the case of the Athens two-year model. One should note, though, that the two-year model

Table 3. Model comparison given as percent errors of predicted from recorded values of the beam transmittance based on 1984 data from Athens, Greece

Month	*Athens 2-YR*	*Randall Whitson*	*Jeter Balaras*
JAN	−5.08	−0.09	0.80
FEB	14.88	17.58	18.43
MAR	13.35	17.18	18.77
APR	4.84	10.50	12.76
MAY*	—	—	—
JUN	−10.93	−3.49	2.35
JUL	−4.47	3.82	7.72
AUG	0.04	7.16	9.27
SEP	−9.70	−2.05	−0.62
OCT	−9.98	−3.86	−6.38
NOV	17.57	14.28	9.22
DEC	−2.04	−0.28	−4.22
Annual Average:	0.77	5.52	6.19

* No data were available for the month of May

derived from the aggregate two year Athens data can still result into relatively large errors for a given month of an annual data base. For example, in November the Athens model overpredicted the recorded values of τ_b by almost 18%. Due to the fact though, that the two-year model (on the average for every month) proportionally overpredicted and underpredicted the recorded τ_b values, its average annual predictive error was kept at a very low level.

The R–W and J–B models typically overestimate the direct radiation in the region of Athens. On a seasonal basis and for specific sky conditions, though, both models were in good agreement with measured data. In particular, we observed that the R–W and J–B models were in best agreement during the fall season (for intermediate to high clearness indices), and the spring season (for low to intermediate k_t cases).

5 CONCLUSIONS

Our objective was to provide some evidence on the performance and applicability of the R–W and J–B models in the area of Athens, and to develop a similar empirical correlation for the region. The proposed two-year model for Athens can be used to decompose available measurements of global radiation to its components, depending on the nature of the application.

Both reference models [7] and [9], according to Figure 1, follow the main body of the data well. However, from the previous statistical results, the R–W and J–B models as expected performed worse when they were statistically compared to the proposed two-year model based on the available 83–85 data from Athens. The linear regression two year model described by Equation (5) and the regression coefficients given in Table 1, can explain 85% of the observed data variation, and on an annual average will predict within 1% the hourly value of the beam transmittance.

Overall, the performance of the Randall–Whitson and Jeter–Balaras models was quite satisfactory. This is an important observation if one takes into account the fact that the three correlations were empirically derived with data from distinctly different geographic regions. It appears then that such models have the prospects of wide applicability, but for Athens, which exhibits atmospheric conditions characteristic of urban regions, the proposed two-year correlation is preferred.

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7 NOMENCLATURE

I_0 solar constant
G hourly global irradiation
I_h hourly direct irradiation
D_h hourly diffuse irradiation

G_0 hourly extraterrestrial horizontal irradiation
I_{0h} hourly extraterrestrial direct irradiation
R prevailing earth-sun distance
R_{ave} average earth-sun distance
x_{0i} lower limit for regression band
y_{0i} regression model at x_{0i}

Greek Characters

β regression coefficient
Δt time interval
δ solar declination
ε residual error
k_t clearness index (ratio of I to I_0)
τ_b beam transmittance (ratio of I_{bn} to I_{0n})
ζ solar zenith angle
Φ latitude
ω solar hour angle

Subscripts

i regression band index
j measured hourly data index

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