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Research Note

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Calculating the surface temperature of the solid underlying surface by modified "Force-Restore" method

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With 2 Figures

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Summary

The calculation of rocky surface temperature encounters the problem of unrealistic results due to its large changes at the interface where the energy balance equation is applied. In order to avoid this, we have modified the "force-restore" method into a self-consistent procedure for simultaneous determination of both surface and the deep ground temperature. The approach is applicable to any celestial body where external radiation can be represented by an arbitrary periodic function. The method is tested with Earth based infrared observation on lunar surface temperature and it showed a high level of accuracy and a rather fast convergence of procedure.

1. Introduction

Rocky surfaces are often the dominant type of ground on the interface between the celestial objects and space or the atmosphere if it exists. That is the reason why, in the atmospheric and other numerical models, the calculation of rock surface temperature should be made with considerable attention. Calculation of the surface temperature, using the energy balance equation at the interface, is more complicated for the rocks than for other solid materials, due to their particular thermal and physical properties.

However, the use of the energy balance equation, for calculating the surface temperature may lead to unrealistic results because of its large changes at the interface where the equation is applied. In order to avoid that, some authors commonly use another approach, the so called "force-restore" method (Stull, 1988), for calculating the solid surface temperature. On the other hand, a very limited number of papers in planetary space science (Stimpson and Lucas, 1972), (Jones et al., 1975), as well as in geophysical sciences (Arsenić and Mihailović, 1995; Mihailović et al., 1996; Arsenić et al., 1997; Arsenić and Mihailović, 1998), is devoted to the rock surface temperature calculation problem.

This paper deals with a modification of the "force-restore" method regarding the estimation of the deep ground temperature T_d and corresponding procedure for the numerical integration of the "force-restore" equation. We have shown theoretically that the suggested procedure is applicable for calculating the surface temperature of any planet or celestial object which has complicated diurnal course of radiation represented by an arbitrary periodic function. Verification of the proposed method is made by comparing our calculated values for the surface temperature of the Moon to values available from the Earth-based infrared measurements.

2. Description of the method

The "force-restore" equation is derived from the energy balance equation (Bhumralkar, 1975) and according to Stull (1988) has the form:

$$C_g \frac{\partial T_g}{\partial t} = R(T_g) - H(T_g) - L(T_g) - \left(\frac{\lambda \omega C}{2}\right)^{1/2} (T_g - T_d) \qquad (1)$$

where: C_g is the surface heat capacity, T_g the surface temperature, $R(T_g)$ the net radiation, $H(T_g)$ the sensible heat flux, $L(T_g)$ the latent heat flux, ω the frequency of diurnal variation of the surface temperature, C the volumetric heat capacity, λ the thermal conductivity and T_d the deep ground temperature which can be estimated either from some prognostic equation or using another method. This equation is applicable for a broad range of the underlying surfaces using an implicit backward scheme for its numerical integration. The basic difference from the original work of Bhumralkar (1975) is that in his approach one does not derive deep ground temperature, but it is taken as a daily average of the surface temperature (more precisely, the temperature of 1 cm thick surface layer). Another difference is the interpretation of terms, because what one usually accepts as "Bhumralkar equation" is obtained after one introduces the temperature as the simple harmonic function of time.

However, Eq. (1) gives reasonable results only in the case when the solid medium contains much liquid water (Mihailović et al., 1998). Then the first three terms on the right hand side of Eq. (1) are significantly greater than the last one, particularly the term representing the latent heat flux. It means that the error in calculation of the last term, so called "force-restore" term, will not affect the final result significantly, because that error will be overshadowed by the comparatively small change in the latent heat flux. Otherwise, if the underlying surface is a rock, the latent heat flux in Eq. (1) may be neglected. Taking into account this fact we will try to calculate the "force-restore" term more generally for any natural solid medium. It will be done by a modification of the "force-restore" method which consists of a new estimation of the deep ground temperature T_d and corresponding procedure for the numerical integration of Eq. (1).

Since the rocky ground has a high level of homogeneity we can assume that the diurnal course of the rock temperature at any depth has the same shape as the diurnal course at the ground surface. Also, it is shifted in time having the amplitude which is smaller than the ground surface temperature amplitude. In order to show it, let us assume that the "diurnal" variation of the surface temperature has the form:

$$T_g(t) = T_s + T_0 F(\omega t) \qquad (2)$$

where $F(\omega t)$ is an arbitrary periodic function with period $T = 2\pi/\omega$. This function can be expanded in Fourier series:

$$T_g(t) = T_s + \sum_{n=1}^{\infty} A_n \cos(n\omega t - \varepsilon_n) \qquad (3)$$

Following Carslaw and Jaeger (1959) (Section 2.6 Eqs. (17) and (18)), the temperature at depth d can be represented as the following series:

$$T_g(t,d) = T_s + \sum_{n=1}^{\infty} A_n e^{-d\sqrt{n}\sqrt{\frac{\omega C}{2\lambda}}} \cdot \cos\left(n\omega t - \varepsilon_n - d\sqrt{n}\sqrt{\frac{\omega C}{2\lambda}}\right) \qquad (4)$$

Usually, one retains only the first term in the Fourier series, i.e. the surface temperature is assumed to have simple harmonic form (Bhumarlkar, 1975, Eq. A2):

$$T_g(t) = T_s + T_0 \cos(\omega t) \qquad (5)$$

leading to

$$T_g(t,d) = T_s + T_0 e^{-d\sqrt{\frac{\omega C}{2\lambda}}} \cos\left(\omega t - d\sqrt{\frac{\omega C}{2\lambda}}\right). \qquad (6)$$

Correspondingly, at the depth d, the daily temperature variations will be $e^{-\varphi}$ times smaller than they are at the rocky surface. Here φ is referred to as the phase shift given by $\varphi = d(\omega C/2\lambda)^{1/2}$.

Using the foregoing assumptions we can calculate the deep ground (rock) temperature T_d

in the following way. In the first iteration we assume that T_d, representing the temperature at a certain depth d, has a constant value i.e., $T_{d,1}^{t_0+i\Delta t} = \text{const.}$. Here, t_0 is the initial time of integration, Δt is the time step, i is the number of time steps, while the second character in the subscript indicates the number (order) of iteration. Further, using this value of $T_{d,1}^{t_0+i\Delta t}$ and applying an implicit backward scheme to Eq. (1), we calculate the rock surface temperature $T_{g,1}^{t_0+i\Delta t}$ as the first iteration. So, in $(n+1)$th iteration, the diurnal course of the deep ground temperature, $T_{d,n+1}^{t_0+i\Delta t}$, is calculated from the previously calculated diurnal course of the rock surface temperature, $T_{g,n}^{t_0+k\Delta t}$, as

$$T_{d,n+1}^{t_0+i\Delta t} = \left(T_{g,n}^{t_0+k\Delta t} - \frac{(1-e^{\varphi})}{N} \sum_{m=1}^{N} T_{g,n}^{t_0+m\Delta t} \right) e^{-\varphi} \tag{7}$$

where N is the total number of time steps during one day of integration, while the number of time steps k in $T_{g,n}^{t_0+k\Delta t}$, depending on the phase shift φ of T_d due to T_g, is related to number of the actual time steps i in $T_{d,n+1}^{t_0+i\Delta t}$ as

$$k = \begin{cases} i - \operatorname{int}\left(\frac{\varphi N}{2\pi}\right) i \geq \operatorname{int}\left(\frac{\varphi N}{2\pi}\right) \\ N + i - \operatorname{int}\left(\frac{\varphi N}{2\pi}\right) i < \operatorname{int}\left(\frac{\varphi N}{2\pi}\right). \end{cases} \tag{8}$$

This procedure is applied until the condition

$$\text{SQRTG} = \left(\sum_{i=1}^{N} \left(T_{g,n+1}^{t_0+i\Delta t} - T_{g,n}^{t_0+i\Delta t} \right)^2 \right)^{1/2} \leq \mu \tag{9}$$

is reached, where μ is a chosen small value.

The computational procedure described above can be successfully applied for calculating the solid surface temperature of any celestial object whose surface is forced by the external radiation which is represented by a periodic function of time (such that retaining the first term of the Fourier series is justified), since we were able to prove that under the above conditions, the "diurnal" course of the surface temperature of the celestial object, consisting of the solid matter, has the same functional form as the "diurnal" course of temperature at any depth.

3. Verification of the method

Verification of the method suggested is made by calculating the lunar surface temperature. There is a physical reason for this choice since the Moon has no exchange of heat by the latent and sensible heat fluxes due to absence of the atmosphere. Thus, on the right hand side of Eq. (1) there remain only two terms, one due to radiation, and other due to the ground heat flux whose imbalance, determined by the errors in estimation of the deep ground temperature, will be more emphasized than in the presence of the atmosphere.

The external forcing by the incoming solar radiation R_i is calculated from the equation

$$R_i = R_0[\sin\Omega \sin\beta + \cos\Omega \cos\beta \sin(\Lambda + \tau)] \tag{10}$$

where R_0 is the solar constant used with the value $1353.0\,\text{Wm}^{-2}$ and the expression in square brackets is the zenith angle of the Sun, while τ and β are the longitude and latitude at the chosen point of lunar surface, Λ is the colongitude of the Sun and Ω is the latitude of the Sun. The values of Λ, Ω are available in the astronomical year books.

The thermal conductivity of the considered lunar surface was derived following the mathematical background and physical constants of paper (Jones et al., 1975), which is partly devoted to heat transfer through the particulate material. The thermal conductivity which strongly depends on temperature is calculated as

$$\lambda(T) = \lambda_0 + \chi_0 T^3 \tag{11}$$

where coefficients λ_0 and χ_0 are approximated by a third order polynomial in terms of the bulk density which is set to be $1970\,\text{kgm}^{-3}$ (an average density in the upper half of the core sample taken by the astronauts in the mission Apollo12). The volumetric heat capacity C, whose dependance on temperature in this study is neglected, is set to be $1.4 \cdot 10^6\,\text{Jm}^{-3}\text{K}^{-1}$. This value, taken from (Horai and Fujii, 1971), represents an average of volumetric heat capacity of the lunar surface, in considered area, consisting mainly of particulate rocks of basalt.

For verification of the method suggested we used the three available data sets with the Earth based observations concerning the lunar surface

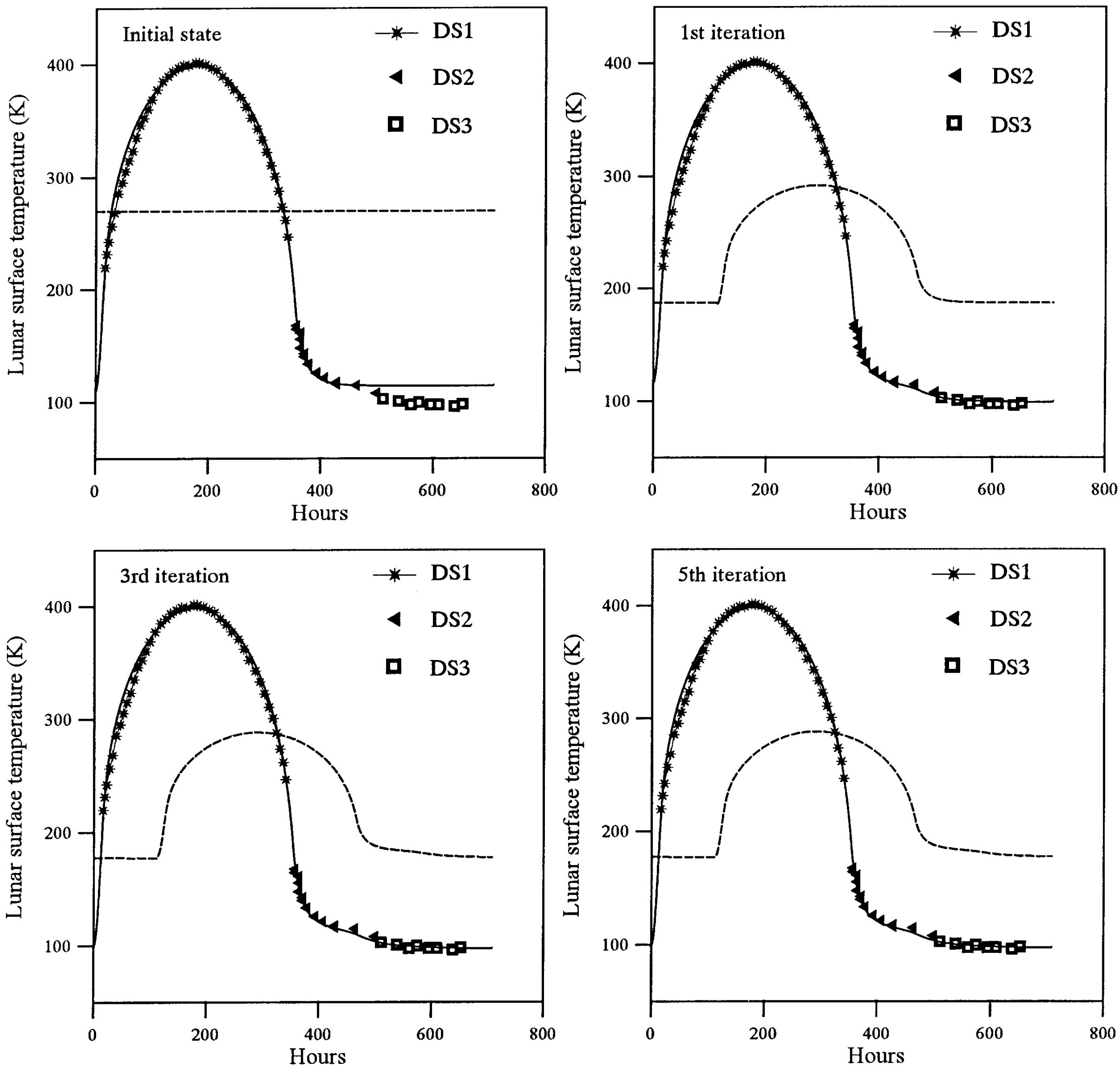

Fig. 1. The calculated "diurnal" variation of the lunar surface temperature (solid line) and the deep ground temperature (dashed line) compared to observations

temperature obtained in the infrared spectral area which, in the further text, will be indicated by DS1 (Stimpson and Lucas, 1972), DS2 (Jones et al., 1975) and DS3 (Jones et al., 1975).

The corresponding numerical model based on the method reported in the preceding section was run with the time step of 360 s and initial conditions for the surface temperature and the deep ground temperature $T_g = 110\,\mathrm{K}$ and $T_d = 280\,\mathrm{K}$, respectively. The calculated "diurnal" variation of the lunar surface temperature is shown in Fig. 1. The solid line represents the surface temperature while the dashed one indicates the "diurnal" variation of the deep ground temperature. By "diurnal" we mean the rotation period of approximately 709 hours. All panels at this figure, represent the evolution of the convergence of the both "diurnal" courses during their passing through the five iterations until the condition (9) is reached.

First of all, a very fast convergence of the calculated values to the observed ones for both curves can be seen in Fig. 1. In fact, the general agreement with the observational data is a achieved practically after the first run, so, one may wonder why further runs are necessary.

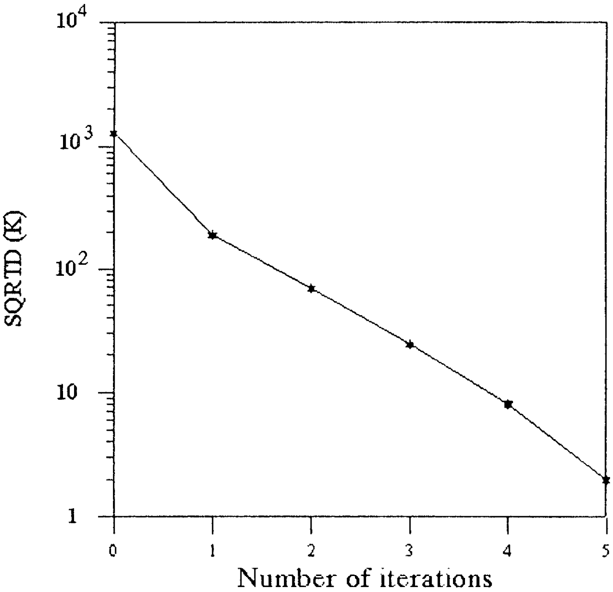

Fig. 2. Evolution of SQRTG defined by the criterion (9)

However, the efficiency of the method can not be completely judged from these plots. The real advantage of the method can be appreciated only from Fig. 2 which shows the evolution of convergence expressed in terms of the lunar surface temperature differences between successive iterations at all computational points given by the expression of the left hand side of inequality (9) Figure 2 indicates that the method suggested practically provides the convergence after the fifth iteration. Moreover, from the point of view of applied earth sciences, where one does not need so strict condition in Eq. (9) as used in this study (3 K), the process of convergence will be even faster. Let us note that 3 K is a total error over 7090 points. A more detailed comparison with other approaches will be published separately.

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