Statistical methods for the prediction of night-time cooling and minimum temperature

G. Emmanouil¹, G. Galanis^{1,2} & G. Kallos¹

¹University of Athens, School of Physics, Division of Applied Physics, University Campus, Bldg. PHYS – V, Athens, 157 84, Greece ²Naval Academy of Greece, Section of Mathematics, Xatzikyriakion, Piraeus 185 39, Greece

Email: gemman@mg.uoa.gr, ggalanis@mg.uoa.gr, kallos@mg.uoa.gr

Four different models for the prediction of frost events caused by air cooling due to thermal radiation are presented in this paper. Three of them are based on polynomial functions that simulate different types of temperature variations in comparison with an exponential one. The fourth, a post-processing method based on Kalman filters, is proposed for those cases where a systematic type of bias has emerged.

Keywords: [still to come].

I. Introduction

Oneof the most important areas in the field of applied meteorology and agricultural applications is the prediction of frost events (i.e. temperatures below 3° C). There are essentially two main frost categories: the first relates to advection of cold air masses and the second to air cooling due to thermal radiation, usually during the night or early hours of the morning.

Cold air advection frost is a result of the general atmospheric circulation. Its duration varies from one to several days and may cause a significant fall in temperature. The main prediction tool for this is numerical atmospheric mesoscale modelling. Thermal radiation frost, which is most frequent in the Mediterranean region, is a consequence of ground cooling at night, usually resulting in a fall of temperature of between $3 \,^{\circ}C$ and $6 \,^{\circ}C$ and is generally observed during late winter and spring. Because of plant growth at this time, the resulting damage can be significant, despite the fact that such frost events last only a few hours.

There are two main ways of forecasting the thermal radiation type of frost event. The first involves high resolution, limited area models, but this requires considerable computing resources in order to obtain acceptable levels of accuracy. The second is based on statistical methods in conjunction with local observational networks. This prediction methodology has been studied by a number of researchers (e.g. Veitch 1958, 1959; Parton & Logan 1981; Gandia et al. 1985; Renquist 1985; Karlsson 2001).

In this paper, we present prediction methods for the thermal radiation frost category. Following the work of Gandia et al. (1985), in which an exponential model is used for the daily prediction of minimum temperature, we propose three different polynomial models which are then compared with the exponential analogue of Gandia et al. This relies on the fact that polynomial functions are able to simulate possible alternations in the type of temperature variation, something not possible for exponential or any other mapping with constant monotonicity.

Additionally, in those cases where the estimation of minimum temperature by means of the above-mentioned methods is systematically biased, we employ a postprocessing method based on Kalman filters, modifying our techniques so as to eliminate systematic errors.

The proposed methodology is easily applied since it does not make heavy demands on computer resources or time, nor uses large statistical datasets. As a matter of fact, it can even be used on micro-processor systems or simple PCs. However, it ensures a very satisfactory quality of temperature prediction, which can be easily adjusted to different weather types. Moreover, it is likely to be a valuable tool in several meteorological applications such as airport/highway monitoring networks, advanced cooling/heating systems, security optimisation against frost and ice or high temperature events.

2. Methodology

Gandia et al. (1985) used an exponential technique in order to predict the minimum temperature, T_m , during a fair night with low wind speeds over the area being Q1 investigated. More precisely, T_m was estimated based on the knowledge of temperature T_b , h hours starting from sunset and to sunrise, and that of sunset T_0 by the formula:

$$\ln\left(\frac{T_b - T_m}{T_0 - T_m}\right) = A_1 \cdot b + A_0. \tag{1}$$

The constant parameters A_0 and A_1 were obtained by means of a least squares adjustment using a large database of past observations.

Despite the fact that the corresponding results presented in their paper were very satisfactory, the above method simulates well the night cooling *only* in those cases where the temperature is decreasing continuously (without any intermediate increment intervals). Such variations, although being very common in nature, cannot be represented by exponential functions due to their constant monotonicity.

Here we propose three different ways of forecasting night cooling in an attempt to overcome the abovementioned drawback, as well as a post-processing method which improves the final output by eliminating any possible standard error.

More precisely, we change first the logarithmic form of (1) to a polynomial one obtaining

$$\frac{T_b - T_m}{T_0 - T_m} = A_0 + A_1 \cdot b + A_2 \cdot b^2.$$
(2)

Using, initially, as in the case of the exponential method, a large database of past observations concerning T_b , T_m and T_0 , we estimate the coefficients A_i (i = 0, 1, 2) by means of a least square regression. In the 22 sequel, the obtained values for A_i are used as constant coefficients for the prediction of T_m by means of (2) given that T_0 , T_b are known at the time of model activation, i.e. h hours after sunrise (T_0). The exact value of *b* is appropriately defined according to seasonal characteristics and depending on the time of sunset. In our study *b* is always considered equal to 3.

Note that the use of this second-order function guarantees not only the capture of the minimum value of temperature, but also, the simulation of any simple inversion of it.

Taking this one step further, we may consider a polynomial of thi rd degree

$$\frac{T_b - T_m}{T_0 - T_m} = A_0 + A_1 \cdot b + A_2 \cdot b^2 + A_3 \cdot b^3, \qquad (3)$$

which can also reproduce inversions of temperature during the night, as well as a possible second minimum value. Such variations could immerge due to temporary changes in local meteorological conditions (e.g. rapid wind speed increase, local cloudiness, etc.). Finally, an even more flexible fourth-order polynomial function is proposed and tested against the previous models. As proved in the sequel of the paper, the Q3 capacity of fourth-order polynomials to simulate time series with two minimal values and intermediate intervals of temperature increment shows this method to be the most accurate.

Using these methods we propose a different way of estimating the minimum temperature that may simulate not only the 'common' exponential temperature decrease during the night but also possible variations in its evolution. The main advantages and disadvantages of the techniques presented are clarified by a detailed comparison study.

As will be shown in the results presented in section 3, in some cases, mainly when the second-degree polynomial or the exponential model is used, a systematic error in the corresponding forecasts emerges. To overcome this, we employ a post-processing method based on Kalman filters. Such filters have proved very effective where the forecast of meteorological parameters is systematically biased. For a detailed study of Kalman filters used for meteorological purposes, see Kalman (1960), Kalman & Bucy (1961), Persson (1991), Brockwell & Davis (1987), Homleid (1995), and Galanis & Anadranistakis (2002). Here, in order to make our argument self-contained, we outline some basic notions.

If an unknown process at time t is represented by the vector \mathbf{x}_t and at the same time a known (*observation*), relevant, vector is denoted by \mathbf{y}_t , then an algorithm estimating the change of the process \mathbf{x} from time t – 1 to t is given by *the system equation*:

$$\mathbf{x}_{t} = \mathbf{F}_{t} \cdot \mathbf{x}_{t-1} + \mathbf{w}_{t}. \tag{4}$$

The relation between the observation vector and the unknown one is described by the *observation equation*:

$$\mathbf{y}_{\mathbf{t}} = \mathbf{H}_{\mathbf{t}} \cdot \mathbf{x}_{\mathbf{t}} + \mathbf{v}_{\mathbf{t}}.$$
 (5)

In both cases the coefficient matrices F_t and H_t , called the *system* and *observation matrix* respectively, must be determined prior to the running of the filter. The same holds for the covariance matrices W_t and V_t of the random vectors w_t , v_t respectively. The latter have to follow the normal distribution with zero mean, must be independent, i.e. $E(w_s \cdot v_t) = 0$ for any $s, t \in N$, and time independent, in the sense that $E(w_s \cdot w_t) =$ $E(v_s \cdot v_t) = 0$, for all $s \neq t$. Here E stands for the mean value factor.

In the present study, the above-described general framework is specified as follows: If we denote by s_t the direct output of the model in use and z_t is the corresponding observation, then we estimate the value

Station ID	Placement	Latitude (deg, min)	Longitude (deg, min)	Height (m)	Data period
16622	Thessaloniki/Micra	40,31	22,58	4	1980–2001
16627	Alexandroupoli	40,51	25,55	3	1980-2001
16648	Larissa	39,38	22,25	74	1980-2001
16684	Skiros	38,54	24,33	18	1980-2001
16710	Tripoli	37,32	22,24	652	1980-2001
16716	Athens/Elliniko	37,54	23,44	15	1980-2001

Table 1. Basic information on the stations in use.

of the relevant error $y_t = z_t - s_t$ by means of a second-order polynomial

$$y_t = x_{1,t} + x_{2,t} \cdot s_t + x_{3,t} \cdot s_t^2$$
.

This order proved to be the optimum choice that combines the most accurate simulation possible of the systematic part of error and, on the other hand, does not create 'noisy' results due to higher order calculations.

The unknown process \mathbf{x}_t is the vector

$$x_t = [x_{1,t} \ x_{2,t} \ x_{3,t}]$$

whose variation in time is given by

$$\mathbf{x}_{\mathbf{t}} = \mathbf{x}_{\mathbf{t}-1} + \mathbf{w}_{\mathbf{t}}.\tag{6}$$

Here \mathbf{w}_t is the non-systematic part of the error in the time variation of \mathbf{x}_t . The coefficient \mathbf{F}_t is assumed to be the unitary matrix since there is no other solid evidence to rely on. We note that the parameter \mathbf{x} does not have specific physical meaning; it is a mathematical tool which helps to obtain a better estimation of temperature minimum values.

The observation matrix H_t takes the form

$$H_t = \begin{bmatrix} 1 & s_t & s_t^2 \end{bmatrix}$$

so the final (corrected) field $scor_t$ at time t is given by

$$scor_{t} = s_{t} + y_{t} = s_{t} + H_{t} \cdot x_{t} = s_{t} + \begin{bmatrix} 1 & s_{t} & s_{t}^{2} \end{bmatrix} \cdot \begin{bmatrix} x_{1,t} \\ x_{2,t} \\ x_{3,t} \end{bmatrix}$$
$$= x_{1,t} + (x_{2,t} + 1) \cdot s_{t} + x_{3,t} \cdot s_{t}^{2}.$$

3. Applications, results and discussion

The first part of this study was concerned with estimating the different coefficients involved in all the techniques used, as described in equations (1)–(3). The observations used in this step were provided by the Hellenic National Meteorological Service (HNMS) for the period 1980–2001. These stations were chosen because they covered the full latitudinal range of Greece, as well as different climatic regions. Some of their basic characteristics are presented in Table 1, while their locations are indicated in Figure 1.

Since our study focuses on night cooling due to thermal radiation, the coefficient estimation was based on the following criteria:

- total cloud coverage less than 3/8,
- wind speed averaging less than 3 m/sec, with a maximum less than 5 m/sec,
- no precipitation observed.

The obtained coefficient values for each station are presented in Tables 3–8.

In the sequel, the methods described in section 2 were Q4 used to forecast minimum temperatures for the period 2002–4 for the same stations. In particular, the choice of the 'appropriate' application days was based on the above-mentioned criteria, which had to be fulfilled at the time of commencement of each method.

A general overview of the performance of the four different methods is presented in Table 2. We used the average absolute error as the evaluation parameter because it best represents each method's deviation.

Table 2. Average absolute error for the six stations and all the methods in use.

Station ID average absolute	166	22	1662	27	1664	18	1668	34	1671	0	1671	.6	Avera	age
error	summer	winter												
4-degree	1.08	1.46	0.97	1.12	1.18	1.09	1.25	0.99	0.95	1.08	0.76	1.00	1.03	1.12
3-degree	3.40	5.35	1.33	1.33	1.77	1.30	1.01	1.22	1.47	1.32	0.96	1.02	1.66	1.92
2-degree	1.10	1.71	3.62	4.41	4.50	6.27	1.63	2.54	4.75	6.69	2.30	3.55	2.98	4.12
exponential	1.13	1.46	0.87	1.24	1.28	2.11	1.05	1.15	1.00	2.38	0.85	2.77	1.03	1.85

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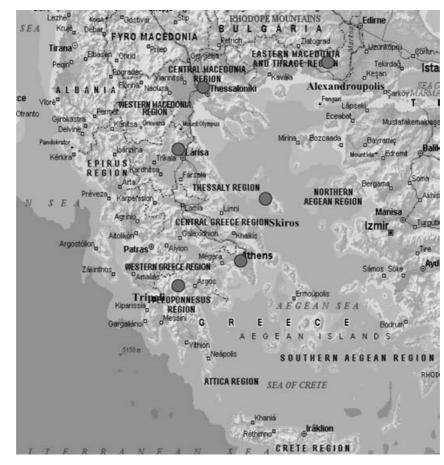


Figure 1. Location of stations in use.

Table 3. Detailed statistical results of	f the different met	thods applied to Thess	aloniki station (16622).

			S	tation 16622				
model	Av. Error	St. dev.	R	A_4	A ₃	A ₂	A_1	A ₀
				Summer				
4-deg	0.83	1.23	0.95	0.0004	-0.0076	0.059	-0.29	1.00
3-deg	3.40	1.65	0.90		0.0011	-0.004	-0.15	0.99
2-deg	0.09	1.46	0.93			0.015	-0.23	1.01
Exp	0.02	1.49	0.92				-0.06	-0.25
				Winter				
4-deg	0.25	2.05	0.91	0.00003	-0.0012	0.022	-0.21	1.00
3-deg	5.35	2.96	0.78		-0.0004	0.015	-0.19	0.99
2-deg	1.52	1.81	0.92			0.006	-0.14	0.96
Exp	0.71	1.93	0.91				-0.06	-0.54

The results are divided into two main time periods: April–October (denoted for convenience as 'summer') and November–March ('winter'). These periods are based on the corresponding main differences in the meteorological synoptic systems that affect the weather fields over Greece as well as differences in day/night duration. As the results show, there are significant differences between the two periods.

In the first period (summer) the proposed fourth-order polynomial procedure and the exponential one proved to be the more accurate. This is mainly due to increased flexibility of a higher order polynomial that better fits the meteorological conditions prevailing during this period, e.g. low wind speeds, relevant absence of precipitation, clear sky – all conditions that favour an increase in radiative cooling as well as possible temperature function's monotony variations.

Q5

In Figures 2 and 3 two such cases are presented. These show that only the fourth-order polynomial method was able to simulate possible intermediate temperature increments or variations that may affect the minimum value. The exponential method fails to 'see' any of them due to its constant monotonicity – a fact that affects its final forecast of minimum value.

Table 4. Detailed statistical results of the different methods applied to Alexandroupoli station (16627).

			9	Station 16627				
model	Av. Error	St. dev.	R	A_4	A ₃	A ₂	A_1	Ao
				Summer				
4-deg	-0.19	1.28	0.95	0.0006	-0.0126	0.090	-0.34	1.00
3-deg	1.26	0.99	0.96		0.0021	-0.017	-0.11	0.99
2-deg	3.62	1.52	0.92			0.019	-0.26	1.02
Exp	0.04	1.22	0.95				-0.02	-0.30
				Winter				
4-deg	0.25	1.47	0.96	0.0001	-0.0026	0.033	-0.23	1.00
3-deg	-0.04	1.87	0.94		-0.0002	0.011	-0.17	0.98
2-deg	4.41	2.75	0.83			0.006	-0.14	0.97
Exp	0.33	1.70	0.94				-0.001	-0.92

Table 5. Detailed statistical results of the different methods applied to Larissa station (16648).

			5	Station 16648				
model	Av. Error	St. dev.	R	A_4	A ₃	A ₂	A_1	A ₀
				Summer				
4-deg	0.36	1.70	0.91	0.0004	-0.0090	0.063	-0.26	1.00
3-deg	1.67	1.58	0.91		0.0017	-0.015	-0.10	0.99
2-deg	4.50	2.34	0.79			0.014	-0.22	1.02
Exp	0.84	1.62	0.91				-0.05	-0.21
				Winter				
4-deg	0.26	1.44	0.96	0.00004	-0.0016	0.024	-0.20	0.99
3-deg	-0.66	1.64	0.95		-0.0003	0.011	-0.17	0.99
2-deg	6.27	4.09	0.67			0.005	-0.13	0.96
Exp	2.03	1.74	0.93				-0.07	-0.42

Table 6. Detailed statistical results of the different methods applied to Skiros station (16684).

			5	Station 16684				
model	Av. Error	St. dev.	R	A_4	A ₃	A ₂	A ₁	A ₀
				Summer				
4-deg	0.02	1.59	0.93	0.0007	-0.0146	0.110	-0.40	1.00
3-deg	0.60	1.12	0.96		0.0019	-0.010	-0.14	0.99
2-deg	1.63	0.93	0.97			0.023	-0.28	1.02
Exp	0.44	1.23	0.95				-0.06	-0.18
				Winter				
4-deg	0.08	1.25	0.94	0.0001	-0.0041	0.053	-0.31	0.99
3-deg	-0.44	1.58	0.91		-0.0005	0.020	-0.22	0.98
2-deg	2.53	1.68	0.89			0.008	-0.15	0.93
Exp	-0.31	1.50	0.91				-0.05	-0.51

In the winter period, the skill of the fourthorder polynomial method is more obvious since it produces better statistical results even compared with the exponential one. Moreover, the third-degree polynomial method shows increased accuracy during this period on account of the fact that early morning increments are delayed due to longer night duration, significant temperature fall (as a result of thermal radiation) and ground heat capacity. Consequently the time of minimum temperature is also delayed and its simulation is more easily fitted to the third-degree polynomial function. In contrast, the second-order polynomial method diverges significantly from the observed values.

In Tables 3–8 we provide a detailed presentation of the results of the four different methods for each station. These results are divided into two time periods ('summer' and 'winter') and include the following evaluation parameters:

- average error of each method (Av. Error),
- standard deviation of Av. Error (Std. Dev.),

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Table 7. Detailed statistical results of the different methods applied to Tripoli station (16710).

			5	Station 16710				
model	Av. Error	St. dev.	R	A_4	A ₃	A ₂	A ₁	A ₀
				Summer				
4-deg	0.12	1.23	0.97	0.0002	-0.0043	0.033	-0.22	1.00
3-deg	1.31	1.31	0.96		0.0012	-0.007	-0.13	0.99
2-deg	4.75	2.72	0.82			0.014	-0.22	1.01
Exp	-0.31	1.30	0.96				-0.06	-0.22
				Winter				
4-deg	0.07	1.47	0.96	0.00003	-0.0011	0.018	-0.18	1.00
3-deg	-0.64	1.68	0.95		-0.0002	0.010	-0.16	0.99
2-deg	6.69	4.32	0.65			0.005	-0.13	0.97
Exp	2.32	1.79	0.93				-0.07	-0.46

Table 8. Detailed statistical results of the different methods applied to Athens station (16716).

			9	Station 16716				
model	Av. Error	St. dev.	R	A_4	A ₃	A ₂	A_1	A ₀
				Summer				
4-deg	0.28	0.93	0.97	0.0003	-0.0048	0.041	-0.25	0.94
3-deg	0.84	0.78	0.97		0.0013	-0.003	-0.15	0.94
2-deg	2.30	1.10	0.95			0.020	-0.25	0.96
Exp	-0.10	1.08	0.95				-0.04	-0.28
				Winter				
4-deg	-0.21	1.27	0.95	0.0001	-0.0019	0.025	-0.21	1.00
3-deg	-0.28	1.29	0.95		-0.0001	0.009	-0.16	0.99
2-deg	3.54	2.15	0.84			0.007	-0.15	0.98
Exp	2.77	1.77	0.89				-0.06	-0.41

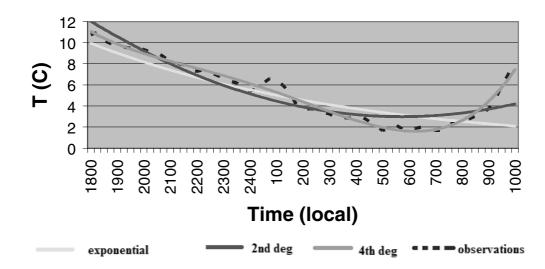


Figure 2. A characteristic time series with intermediate temperature rise during the night.

 correlation coefficient of the linear regression line (y = Ax + B) between observations x and model outputs y,

4, 3, 2), and the coefficients A₀, A₁ of relation (Eq. 1) for the exponential method.

• regression coefficients $(A_n, A_{n-1}, ..., A_1, A_0)$ of each polynomial method, resulting in a function $p_n(x) = A_n x^n + A_{n-1} x^{n-1} + ... A_1 x + A_0$ (n= From this analysis, it is obvious that during 'summer' the fourth-degree polynomial model, having an average error very close to zero, simulates the temperature

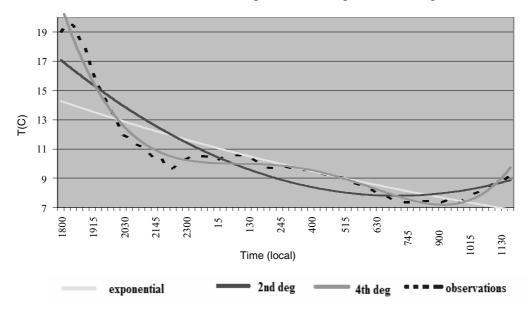


Figure 3. A case with two temperature minima.

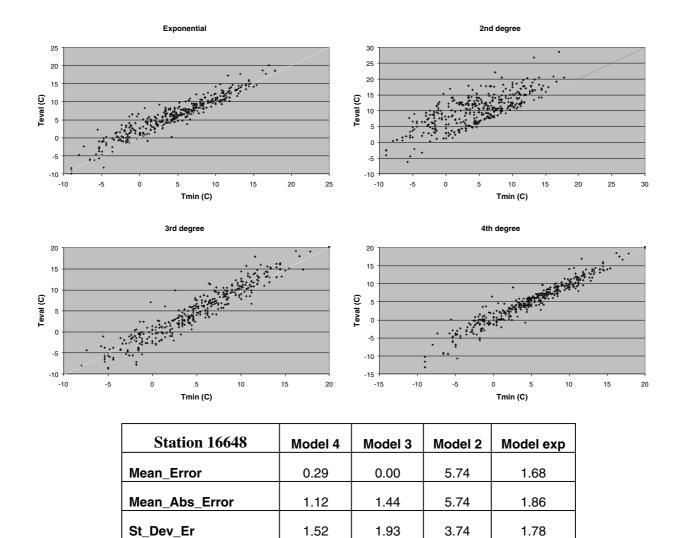


Figure 4. Station 16648: xy-scatter diagram of minimum observed and evaluated temperature for all methods in use. We use T1 y = x as the regression line (yellow). Basic statistical parameters are also presented.

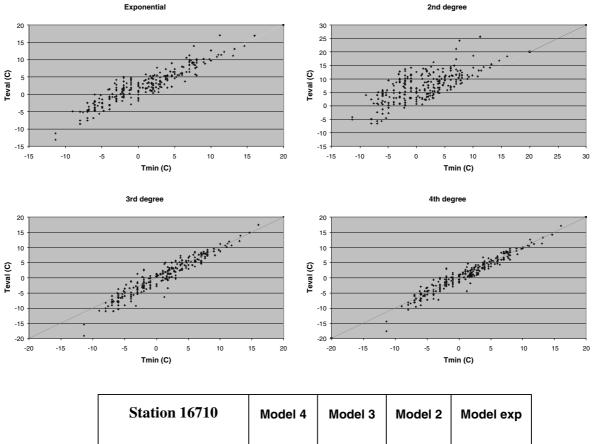
1.93

6.85

2.44

1.54

Root_Mean_Sq_Error



Station 16710	Model 4	Model 3	Model 2	Model exp
Mean_Error	0.08	-0.27	6.28	1.81
Mean_Abs_Error	1.05	1.33	6.28	2.11
St_Dev_Er	1.41	1.77	4.12	1.98
Root_Mean_Sq_Error	1.41	1.78	7.50	2.68

Figure 5. Station 16710: xy-scatter diagram of minimum observed and evaluated temperature for all methods in use. We use T1 y = x as the regression line (yellow). Basic statistical parameters are also presented.

decrease with almost no systematic error. However, taking into account all the available evaluation indices, we may conclude that the exponential method also surpasses the other two and especially the second-order polynomial, whose performance is rather poor.

The same conclusions hold for the 'winter' period too, but with higher quality results for the fourthorder technique and increased noise in the second order, mainly due to the longer night duration, as well as the significant temperature decrease and ground heat capacity. These all relate to the delay in the time when the minimum temperature occurs.

The above results are further outlined in the x - y scatters of Figures 4 and 5 for the three different cases. The correlation of the results with the regression line (y = x), as well as the corresponding statistical results relating to average error and its absolute value, again

confirm the dominance of the fourth-order polynomial method in all these test cases.

Going one step further, for those cases where systematic errors emerged, we applied the post processing method based on Kalman filters described in section 2. The corresponding results for the test cases mentioned above are presented in the time series shown in Figures 7–12. T2

The improvement of the final forecast after using the proposed post-processing method is impressive, especially for these time periods where a standard type of divergence occurs. This is also shown in Tables 9– 11 where some characteristic statistical parameters are presented. The success of the Kalman filter is guaranteed by the elimination of average error in each case studied. In all these cases the final results are comparable to those of the fourth-degree polynomial method. The use of any post-processing method of this type on the latter is meaningless due to the small average bias of its results.

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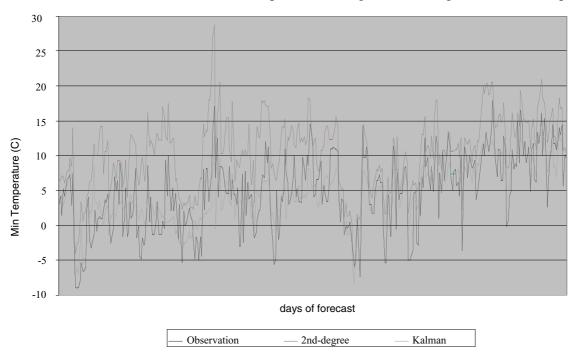


Figure 6. Station 16648: Observations against second-order approach and Kalman filter.

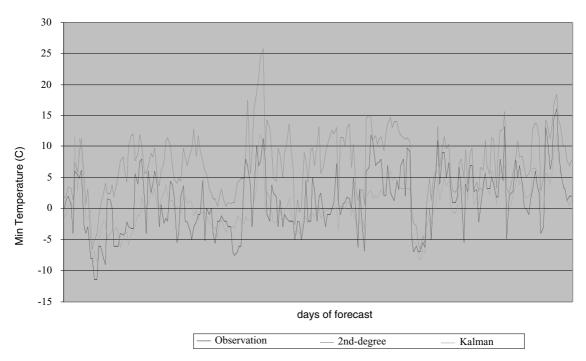


Figure 7. Station 16710: Observations against second-order approach and Kalman filter.

Table 9. Comparison of fourth-degree polynomial, exponentialand Kalman filtered exponential models for 16710 station.

Table 10. Comparison of fourth-degree polynomial, second-degree and Kalman filtered second-degree models for 16710 station.

Station 16710	Av. Error	Av. Abs. Error	Stand. Dev.	Station 16710	Av. Error	Av. Abs. Error	Stand. Dev.
4-deg	0.09	1.05	1.41	4-deg	0.09	1.05	1.41
Exp + Kalman	-0.23	1.43	1.80	2-deg + Kalman	-0.60	2.98	3.71
Exp	1.80	2.10	2.00	2-deg	6.26	6.26	4.11

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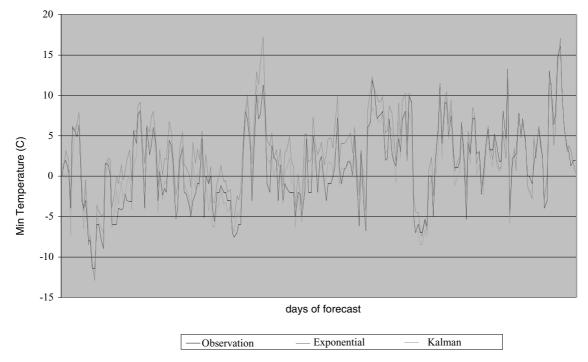


Figure 8. Station 16710: Observations against exponential approach and Kalman filter.

Table 11. Comparison of fourth-degree polynomial,
second-degree and Kalman filtered second-degree models for
16648 station.

Station 16648	Av. Error	Av. Abs. Error	Stand. Dev.
4-deg	0.29	1.12	1.52
2-deg + Kalman	-0.20	2.61	3.27
2-deg	5.74	5.74	3.75

4. Conclusions

Four different methods of predicting minimum temperature were presented and applied to a number of locations covering a wide latitudinal range across Greece as well as different climatological conditions. We used three polynomial-based methods (of second, third and fourth degree) as well as an exponential one. The corresponding results were verified against observations provided by the Hellenic National Meteorological Service's stations in two different seasons: 'summer' (April–October) and 'winter' (November–March). The main conclusions are as follows:

- During 'summer', the fourth-order polynomial method as well as the exponential one gave more accurate results.
- During 'winter', the prevalence of the fourth-order polynomial is further sustained since it is the only one able to simulate alternations of temperature variation during night as well as a possible fall after sunrise.

- The second-degree method gave low quality forecasts in all cases, especially during the 'winter' period.
- The post-processing Kalman filter method, applied operationally in those cases where systematic divergences from the observed values emerged, led to significant improvement of the final results, which were comparable to those obtained by the (optimal) fourth-degree polynomial.

The use of the methodology presented here makes possible not only an accurate prediction of minimum temperature but also the time when the this occurs, thus providing a very valuable tool for the prediction and mitigation of frost events, requiring only minimal computing resources.

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